

Composite magnetic dark matter and the 130 GeV line

James M. Cline^{*} and Guy D. Moore[†]

Department of Physics, McGill University, 3600 rue University, Montréal, Québec, Canada H3A 2T8

Andrew R. Frey[‡]

*Dept. of Physics and Winnipeg Institute for Theoretical Physics,
University of Winnipeg, Winnipeg, MB, R3B 2E9, Canada*

We propose an economical model to explain the apparent 130 GeV gamma ray peak, found in the Fermi/LAT data, in terms of dark matter (DM) annihilation through a dipole moment interaction. The annihilating dark matter particles represent a subdominant component, with mass density 7 – 17% of the total DM density; and they only annihilate into $\gamma\gamma$, γZ , and ZZ , through a magnetic (or electric) dipole moment. Annihilation into other standard model particles is suppressed, due to a DM mass splitting in the magnetic dipole case, or to p -wave scattering in the electric dipole case. In either case, the observed signal requires a dipole moment of strength $\mu \sim 2/\text{TeV}$. We argue that composite models are the preferred means of generating such a large dipole moment, and that the magnetic case is more natural than the electric one. We present a simple model involving a scalar and fermionic techniquark of a confining $SU(2)$ gauge symmetry. We point out some generic challenges for getting such a model to work. The new physics leading to a sufficiently large dipole moment is below the TeV scale, indicating that the magnetic moment is not a valid effective operator for LHC physics, and that production of the strongly interacting constituents, followed by technihadronization, is a more likely signature than monophoton events. In particular, 4-photon events from the decays of bound state pairs are predicted.

In the past few months a number of studies of publicly available Fermi Large Area Telescope [1] data have emerged that find evidence for a spectral peak of energy ~ 130 GeV from the galactic center [2–4], possibly accompanied by a second peak with lower energy ~ 116 GeV [5, 6]. Ref. [7] moreover finds corroborating evidence from outside the galaxy, originating from several galactic clusters, and ref. [8] sees marginal evidence for sources unassociated with known structures, though this has been questioned [9–11]. The morphology of the signal from the galactic center is consistent with dark matter annihilating into monoenergetic photons, but not with dark matter decays [7, 12]. In terms of the thermally averaged cross section required for the standard DM relic density, $\langle\sigma v\rangle_0 = 3 \times 10^{-26} \text{cm}^3/\text{s}$, the annihilating DM hypothesis is consistent with a cross section estimated to be $0.04 \langle\sigma v\rangle_0$ [3, 12] or $0.1 \langle\sigma v\rangle_0$ [4, 7]. Fermi/LAT itself places a limit of $\langle\sigma v\rangle_{\gamma\gamma} < 0.035 \langle\sigma v\rangle_0$ on this cross section (for an Einasto profile) [13, 14], marginally below the required value. Searches for the signal from dwarf galaxies yield the weaker limit $\langle\sigma v\rangle_{\gamma\gamma} < 1.3 \langle\sigma v\rangle_0$ [16]. If the 130 GeV line is accompanied by a gamma continuum due to additional annihilation channels into charged particles, the limit on the cross section for these processes is estimated by ref. [15] to be $\langle\sigma v\rangle_{\text{other}} < 3.5 \langle\sigma v\rangle_{\gamma\gamma}$ for the $b\bar{b}$ channel, while ref. [17] obtains the weaker limit $\langle\sigma v\rangle_{\text{other}} < 20 \langle\sigma v\rangle_{\gamma\gamma}$. (Weaker limits are found in ref. [18], and stronger ones in [19]). While the latter is only

marginally in conflict with the DM getting its standard relic density from annihilating primarily into charged particles, the former would definitely forbid this scenario, presenting a challenge to many models.

Although some authors have expressed skepticism about the DM interpretation of the 130 GeV line [20, 21], there has been a great deal of interest in this possibility both from the experimental analysis perspective [22]–[26] and in theoretical model-building [27]–[36]. (Ref. [37], which preceded the observation of the 130 GeV line, is also a competing theory.) In the present paper, we consider a distinctive model similar to those discussed in refs. [38, 39], in which the dark matter interacts primarily through a large magnetic dipole moment (MDM). Direct detection of magnetic dark matter has been the focus of many recent papers [40]–[49] (for recent work on indirect detection, see [50]). Here we show that Majorana DM with a transition MDM and a relatively large mass splitting can economically explain the observed gamma ray line, in particular if we relax the usual assumption that the candidate particle must be the dominant form of dark matter. (As we will discuss, DM with an electric dipole moment (EDM) and a smaller mass splitting can also work, though we do not favor this scenario.) Moreover we remove the tension of too large a γ -ray continuum by arranging for the 2γ annihilation channel (along with the accompanying γZ and ZZ channels) to be the dominant one. Majorana DM χ_1 can have a transition magnetic moment to a heavier state χ_2 ,

$$\frac{1}{2}\mu_{12}\bar{\chi}_1\sigma_{\mu\nu}\chi_2 F^{\mu\nu}. \quad (1)$$

It is natural to assume that the interaction (1) came originally from standard model hypercharge, and is therefore accompanied by a corresponding coupling to the Z boson

^{*}Electronic address: jcline@physics.mcgill.ca

[†]Electronic address: guymoore@physics.mcgill.ca

[‡]Electronic address: a.frey@uwinnipeg.ca

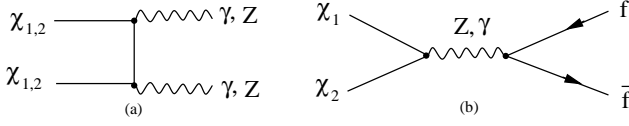


FIG. 1: Annihilation processes for determining the relic density. (a) Annihilations $\chi_{1,2}\chi_{1,2} \rightarrow \gamma\gamma$ (the u -channel diagram with crossed initial or final states is not shown); (b) coannihilations $\chi_1\chi_2 \rightarrow f\bar{f}$ where f is a charged SM particle (or Higgs in the case of Z intermediate state).

field strength.

With the interaction (1), there exist both annihilations $\chi_{1,2}\chi_{1,2} \rightarrow \gamma\gamma, \gamma Z, ZZ$, as well as coannihilations $\chi_1\chi_2 \rightarrow f\bar{f}$, where f is any charged standard model particle, or hZ in the case of the Z intermediate state. These are pictured in fig. 1. As was shown in ref. [39], the coannihilations suppress the relic density below its required value; but a mass splitting $m_{\chi_2} - m_{\chi_1} \sim 10$ GeV suppresses the coannihilations enough to get the right relic density. Therefore a rather large mass splitting is a needed ingredient in the magnetic dipole case. Such a large mass splitting of course makes direct detection of χ_1 from scattering on nuclei impossible, because of the large inelasticity. On the other hand, if the coupling is via an electric dipole moment, which has an extra factor of γ_5 , then the troublesome annihilations into SM particles are p -wave suppressed and can be ignored.

In any case, there is value of μ such that the observed line intensity is achieved, allowing for the relic density to be smaller (by a factor of ~ 10) than the standard value, and then χ is a subdominant component of the total DM. We do not concern ourselves here with the identity of the majority of dark matter, although it is interesting to note that it could be for example a ~ 10 GeV particle as suggested by observations of the CoGeNT and CRESST experiments.

We will assume that the DM carries no $SU(2)_L$ charge, but only weak hypercharge. In that case the interaction (1) is accompanied by the analogous coupling to the Z boson field strength, suppressed by $\tan\theta_w \equiv t_w$:

$$-\frac{1}{2}t_w\mu_{12}\bar{\chi}_1\sigma_{\mu\nu}\gamma_5\chi_2 Z^{\mu\nu} \quad (2)$$

The relative sign between (1) and (2) comes from the relation between hypercharge and the mass eigenstates, $B^\mu = c_w A^\mu - s_w Z^\mu$.

Determining μ and relic abundance. We assume that the dominant annihilation channels are to $\gamma\gamma, \gamma Z, ZZ$ via a dipole moment, as shown in fig. 1(a)—both in the context of current galactic dark matter signals, as well as that of the annihilations which establish the relic abundance. Let us call the cross-section to two photons $\langle\sigma_{\gamma\gamma}v\rangle$; then using Eq. (2), the total cross-section is $\langle\sigma v\rangle \simeq \langle\sigma_{\gamma\gamma}v\rangle/\cos^4\theta_w$ (neglecting corrections from the different kinematics of a Z final state, see appendix

B). We also write the density of 130 GeV dark matter as n , which will be smaller than the standard relic density n_0 . The total 130 GeV photon production rate scales as $\langle\sigma_{\gamma\gamma}v\rangle n^2$. According to refs. [3, 4] the observed γ ray signal corresponds to

$$\langle\sigma_{\gamma\gamma}v\rangle n^2 = (0.04-0.1)\langle\sigma v\rangle_0 n_0^2, \quad (3)$$

where $\langle\sigma v\rangle_0$ is the cross-section which would lead, via the usual freezeout calculation, to a relic density of n_0 . Since the relic density is approximately inversely proportional to the annihilation cross-section,

$$n \simeq \frac{\langle\sigma v\rangle_0}{\langle\sigma v\rangle} n_0, \quad (4)$$

the photon producing cross-section should satisfy

$$\frac{\langle\sigma_{\gamma\gamma}v\rangle}{\langle\sigma v\rangle_0} \simeq \frac{\cos^8\theta_w}{(0.04-0.1)}. \quad (5)$$

Restoring the Z -mass corrections found in appendix B, we find that the dipole moment should lie in the range

$$1.6 \text{ TeV}^{-1} < \mu_{12} < 2.0 \text{ TeV}^{-1} \quad (6)$$

and the fractional relic abundance is

$$0.07 < n/n_0 < 0.17. \quad (7)$$

Notice that a larger dipole moment leads to more complete annihilation during freezeout, and hence to a smaller abundance and a smaller photon flux.

If another annihilation mechanism was active in the early Universe—such as coannihilation, which can occur if the χ_1 to χ_2 mass splitting is not too large [39]—then $\langle\sigma_{\gamma\gamma}v\rangle$ and hence μ_{12} need to be smaller, and the relic density fraction is larger. In the extreme case that coannihilation controls the relic density and $n = n_0$, then $\langle\sigma_{\gamma\gamma}v\rangle$ can be about 100 times smaller, and hence μ_{12} can be 10 times smaller. The coannihilation cross section is just large enough to give the right relic density (if the mass splitting is small) for this smaller value of μ_{12} .

Direct detection. If the DM candidate χ was Dirac, an elastic EDM or MDM as large as in Eq. (6) would be ruled out by many orders of magnitude, by direct detection searches for elastic scattering on nuclei. In ref. [44] (see also [41]-[43],[45]-[49]) it was shown that by splitting the Dirac particle into two Majorana states with mass splitting of ~ 150 keV, a magnetic moment $\sim 10^{-2}\mu_N = 1.6/\text{TeV}$ is compatible with direct detection limits (though somewhat too large to make the DAMA annual modulation signal compatible with these bounds). Depending upon the nucleus, the scatterings can be dominated by dipole-dipole or dipole-charge interactions between the DM and the nuclei, both of which are velocity-suppressed (compared to EDM) in the case of MDM.

The above determination was made assuming that the dipolar DM had the standard relic density; in our case

the density is lower by about a factor of 10. Thus our DM candidate looks similar to DM with the full relic density but a smaller magnetic moment, reduced by the factor $\sim \sqrt{10}$. This reduction by half an order of magnitude is enough to be marginally compatible with fits to the DAMA data, with mass splitting ~ 125 keV; see fig. 2 of [44]. However we cannot reconcile such a small mass splitting with the need to suppress the $\chi_1\chi_2 \rightarrow f\bar{f}$ coannihilation channels.

On the other hand, for EDM dark matter, the coannihilations are p -wave suppressed even if the mass splitting is small, which suggests the possibility of combining the EDM explanation of the 130 GeV line with direct detectability. EDM scattering is dominated by interaction with the charge of the proton, and is enhanced by $1/v^2$ relative to scattering of an MDM (see the erratum of ref. [40]). For inelastic scattering with mass splitting δM , we find that this translates to an enhancement by the factor $m_n/\delta M$ in the cross section. Therefore one would have to reduce the EDM by a factor of $\sqrt{m_N/\delta M} \sim 100$ relative to the above estimate for it to be relevant for direct detection, which would then make it too small to explain the 130 GeV line. We find the same conclusion whether dipole-dipole or dipole-charge scattering dominates in the MDM interaction. (Notice that one cannot increase δM much above 100 keV for direct detection since the minimum required DM velocity would then exceed the escape velocity of the galaxy.)

Model requirements. Consider first that the dark matter is a neutral fundamental field χ . One way it can obtain a large dipole moments is through a Yukawa interaction $y\bar{\chi}\phi S$ to heavy (hyper)charged states ψ , S . If all the states have masses ~ 100 GeV, then an EDM of order $1/\text{TeV}$ is only achieved by pushing y to the perturbative limit $\sqrt{4\pi}$ and tuning the particles in the loop to be nearly on shell [51]. Large MDMs require similar tuning of parameters. Thus any perturbative origin requires stretching the limits of perturbation theory; probably the effective theory describing such states cannot be valid much higher than the particle mass scales.

On the other hand, large magnetic moments are known to arise for neutral composite particles. The neutron is a good example, with $\mu = -1.91(e/2m_p)$, which is approximately the sum of the magnetic moments of the constituent quarks (treating them to have constituent quark masses $\sim m_N/3$, see for example ref. [52]). If the DM is a bound state and analogously $\mu \sim e/2m_\chi$, this gives $1.1/\text{TeV}$, which has the right order of magnitude. This could happen if the hidden sector has a confining gauge symmetry such as $\text{SU}(3)$, analogous to QCD, that becomes strong at the 100 GeV scale. However we cannot push the analogy to QCD too far. Not only do we need the DM particle to be absolutely stable, we need it to be the lightest bound state, since otherwise it would have a large annihilation cross-section into lighter bound states. Therefore it cannot be a baryon of $\text{SU}(N_c > 2)$, since there will always be lighter mesonic states.

There will however be fermionic meson states in theories with both scalar and spinor matter, say a charged scalar techniquark S and an oppositely-charged anti-techniquark ψ forming a spin- $\frac{1}{2}$ bound state $\eta = S\psi$. Such models automatically also have composite neutral bosonic mesons $\tilde{\eta}_S = S^*S$ and $\tilde{\eta}_\psi = \bar{\psi}\psi$. If the η is lighter than the other mesons and baryons, it is safe from annihilating into them; and if the S, ψ mass scales are below or comparable to the confinement scale, the glueballs are also heavier. The details of the strong dynamics may determine which meson is the lightest, but we argue in the next section that it can plausibly be the η .

A further requirement is to split the Dirac η state into Majorana states of different mass, in order to have a transition dipole moment. In the theoretically preferred scenario of large magnetic moment, this splitting has to be sizable, $\gtrsim 10\%$, to get a sufficient relic density. A loop effect (such as that which gives the neutron a mass splitting in R -parity violating supersymmetry [53, 54]) will be too small. But there is a simple way to get a large mass splitting at tree level, if the dark matter is an admixture of an elementary fermion χ and the bound state $S\psi$, by having a bare mass term $\frac{1}{2}m_\chi\bar{\chi}\chi$ and a Yukawa coupling $y\bar{\chi}\psi S$. The latter becomes an off-diagonal mass term with mass of order $y\Lambda$ at scales below the confinement scale Λ . Large mass splittings result as long as m_χ is comparable to m_η , the (unmixed) bound state mass. The generation of the transition moment can be visualized through the diagrams of fig. 2.

Finally, the exotic charged states that appear as bound states (or within the loop for a perturbative origin of the dipole) must not be stable since there are very stringent constraints on charged relics. Thus any model introducing exotic heavy charged particles to induce the dipole moment must ensure that they can decay into charged particles within the standard model, while respecting the stability of the dark matter.

Explicit model. Based on the above considerations, we construct the simplest model of composite dipole dark matter¹ that meets all the requirements. We take the new confining gauge group to be $\text{SU}(2)_g$, and introduce a fundamental $(\text{SU}(2)_g \text{ doublet})$ Dirac fermion ψ and complex scalar S that each carry electric charge $1/2$. There is an elementary Majorana fermion χ that mixes with the neutral $S\psi$ bound state to form the dark matter. The particle content and quantum numbers are listed in table I.

The relevant mass terms and interactions in the potential at the scale above Λ (the compositeness scale) are

$$V = \frac{1}{2}m_\chi\bar{\chi}\chi + m_\psi\bar{\psi}\psi + m_S^2|S|^2 + \lambda|S|^4 + \bar{\chi}S^a(y + iy_5\gamma_5)\psi_a + y'\epsilon_{ab}S_a^*\bar{e}_R\psi_b + \text{h.c.} \quad (8)$$

¹ For another recent model of composite DM, without a dipole interaction however, see ref. [55]

state	spin	SU(2) _g	U(1) _y	U(1) _{em}	Z ₄	constituents
χ	$\frac{1}{2}$	1	0	0	-1	-
ψ_a	$\frac{1}{2}$	$\bar{2}$	-1	$-\frac{1}{2}$	i	-
S^a	0	2	1	$\frac{1}{2}$	i	-
η	$\frac{1}{2}$	1	0	0	-1	$S\psi$
$\tilde{\eta}_S$	0	1	0	0	1	S^*S
$\tilde{\eta}_\psi$	0	1	0	0	1	$\bar{\psi}\psi$
N^-	$\frac{1}{2}$	1	-2	-1	1	$S^*\psi$
\tilde{N}_μ^+	0	1	2	1	-1	SS
\tilde{N}_ψ^-	0	1	-2	-1	-1	$\psi\psi$

TABLE I: Particle content and representations of the confining gauge group, weak hypercharge, electric charge, discrete global DM number symmetry charge, and particle content (in the case of composite states), for states in the minimal model of composite dipole dark matter. Top 3 rows are the elementary constituents, bottom rows are the mesonic and baryonic bound states.

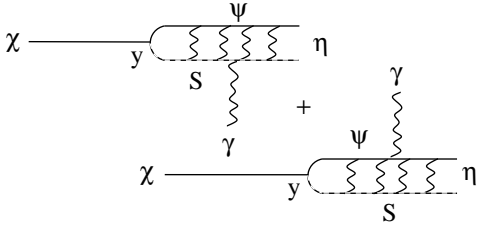


FIG. 2: Schematic Feynman diagrams for generation of large transition dipole moment between the elementary state χ and the composite one η .

The interactions are invariant under a discrete Z_4 symmetry, shown in table I, which guarantees the stability of the dark matter. The last interaction involves electrons for simplicity, but in general any combination of right-handed charged leptons can appear.

The physical states of the system are the neutral mesonic bound states $\eta = S^a\psi_a$, $\tilde{\eta}_S = S_a^*S^a$, $\tilde{\eta}_\psi = \bar{\psi}^a\psi_a$, and the charged baryonic² ones $N^- = \epsilon_{ab}S_a^*\psi_b$, $\tilde{N}_\mu^+ = \epsilon_{ab}S^a\partial_\mu S^b$, $\tilde{N}_\psi^- = \epsilon_{ab}\psi_a\psi_b$. The state η will be the lightest for some part of the parameter space, since $\tilde{\eta}_S$ gets a positive mass correction from the repulsive quartic scalar interaction³, there is a positive spin-spin interaction energy for $\tilde{\eta}_\psi$, and we are still free to choose m_χ, m_S – which however cannot be much heavier than the confinement scale so that $m_\eta < m_{\text{glueball}}^{0++}$. The charged states

are heavier due to positive Coulombic energy shifts.

The bound state η mixes with χ through the first Yukawa interaction in (8). The mass terms relevant for the DM and its excited states are

$$\frac{1}{2}m_\chi\bar{\chi}\chi + m_y(\bar{\chi}e^{i\theta_y\gamma_5}\eta + \text{h.c.}) + m_\eta\bar{\eta}\eta \quad (9)$$

where m_y is of order $(y^2 + y_5^2)^{1/2}\Lambda$, $\theta_y = \tan^{-1}(y_5/y)$, and $m_\eta \sim \Lambda$ is the mass of the η composite fermion. For simplicity, we impose parity to set $\theta_y = 0$ in the following, though the qualitative features of the model do not depend upon this choice. In the basis of Weyl components (η^c , η , χ) of a single handedness, this corresponds to a 3×3 Majorana mass matrix of the form

$$M = \begin{pmatrix} 0 & m_\eta & m_y \\ m_\eta & 0 & m_y \\ m_y & m_y & m_\chi \end{pmatrix}. \quad (10)$$

As long as $m_y \neq 0$, the mass eigenstates are distinct Majorana particles, none of which can be paired up into a Dirac particle. This is important, since as noted above, a stable Dirac particle with as large a direct magnetic moment as $\mu_\eta = 1/\text{TeV}$ is ruled out by direct detection constraints. Notice that in the same basis, the magnetic moment matrix takes the form

$$\mu = \begin{pmatrix} 0 & \mu_\eta & 0 \\ -\mu_\eta & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (11)$$

Thus if $R^T M R$ diagonalizes the mass matrix, then $R^T \mu R$ gives the transition moments in the mass eigenbasis.

In the parity-conserving case, the spectrum of DM states has a simple analytic form, which we give in appendix C. The combination $(\eta - \eta^c)/\sqrt{2}$ does not mix with χ and retains mass of exactly m_η . For small m_y , the spectrum can be written in the form

$$m_i = (m_\eta - \delta m, m_\eta, m_\chi + \delta m) \quad (12)$$

where $\delta m \cong 2m_y^2/(m_\chi - m_\eta)$. We argue that to avoid suppressing μ_{12} by a small mixing angle, it is desirable to choose $m_\eta < m_\chi$ so that the lowest two states are mostly η, η_c . Then the mixing angle between the lowest and highest states is $\theta_{13} \cong \sqrt{2}m_y/(m_\chi - m_\eta)$, and the middle state is purely composite; if $m_\chi < m_\eta$, the masses take the inverse hierarchy $m_3 < m_2 < m_1$. The transition moments in either case are $\mu_{12} \simeq \cos\theta\mu_\eta \simeq \mu_\eta$ and $\mu_{23} \simeq \theta\mu_\eta$. By analogy to the nucleons of QCD, we can estimate $\mu_\eta \simeq (e/2)/(2m_\eta/2) \simeq 1.2/\text{TeV}$. Based on the quantitative success of the quark model in reproducing the baryon magnetic moments [52], this can be considered as good to $\sim 10\%$, and not just an order of magnitude estimate. Comparing to the needed value in Eq. (6), we see that the transition moment involving the lightest state will be large enough provided that the lowest mass state is predominantly η , which leads us to choose

² For SU(2) there is no real distinction between mesons and baryons; so our choice to call these baryons is just a convention.

³ There is a potential danger that λ runs to negative values before the confinement scale, due to a g^4 term in its beta function β_λ . This is partly compensated by the $-y^4$ term in β_λ . Here we will assume that parameters can be found where it does not go negative below the scale m_S .

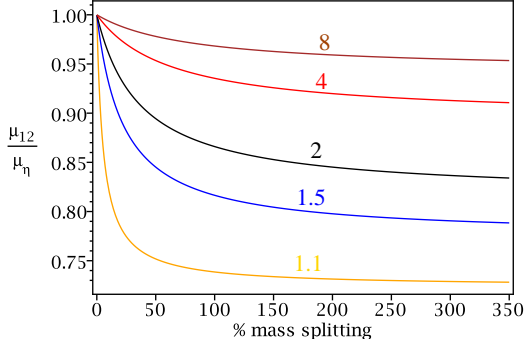


FIG. 3: Ratio of transition moment μ_{12} to its maximum value μ_η as a function of the relative mass splitting between the DM and its first excited state, for $m_\chi/m_\eta = 1.1, 1.5, 2, 4, 8$, as labeled.

$m_\eta < m_\chi$. We also need a small amount of coannihilation, somewhat reducing the required magnetic moment from the one presented in Eq. (6).⁴

We thus arrive at the favored scenario in which $m_\chi > m_\eta > m_y$, with $\delta m/m_\eta \equiv r \sim 10\%$ so coannihilations are suppressed but not quite absent. We have computed the ratio of the transition moment μ_{12} to its maximum value μ_η as a function of r , which is plotted as a function of r for several values of m_χ/m_η in fig. 3. The exact mass splittings and mixing angles are given in appendix C in this parity-symmetric case. (We leave the parity-violating case for future work; generally, all three states will mix, and none of the possible transition moments will vanish.)

The baryonic bound states carry electric charge, and must not be stable because of very stringent bounds on relic charged particles. The y' Yukawa coupling ensures that they can decay into a charged lepton ℓ^\pm and a photon or dark matter. For the N^- baryon, the y' coupling⁵ in (8) leads directly to a mass mixing term $\tilde{y}'\Lambda \bar{e}_R N^-$, so that in the mass eigenstate basis, an off-diagonal coupling $\bar{e}_R A N^-$ is induced, allowing the decay $N^- \rightarrow \gamma \ell^-$. Because of their nontrivial Z_4 charges, the bosonic baryons must decay via $\tilde{N}_\mu^+ \rightarrow \eta \ell^+$ and $\tilde{N}_\mu^- \rightarrow \eta \ell^-$ respectively, through the elementary processes $S \rightarrow \psi \ell^+$ or $\psi \rightarrow S \ell^-$.

⁴ An alternative way to increase the magnetic moment is to choose the electric charge of the constituents to be larger. For example taking $\pm 3/2$ instead of $\pm 1/2$ increases the magnetic moment by a factor of 3. The decay of charged relics in this case can be mediated by the dimension-7 operator $y'\epsilon_{ab}S_a^*\bar{e}_R\psi_b(\bar{e}_R e_R)/M^3$ with some large mass scale M .

⁵ Searches for lepton compositeness constrain $\Lambda_c \gtrsim 10$ TeV in operators like $(\bar{e}e)^2/\Lambda_c^2$, induced by a box diagram with virtual ψ and S , giving the weak limit $y' \lesssim (100 \text{ GeV}/10 \text{ TeV})^{1/2} = 0.1$. Such operators are also the leading source of leptonic FCNC's when we consider y'_i couplings to all generations of leptons, since the induced dimension-4 operator, going like $y'_i y'_j/(16\pi^2) \bar{e}_{R,i} i \not{\partial} e_{R,j}$, is just wave function renormalization that can be removed by a redefinition of the lepton Yukawa matrix.

Clearly, only a small mass splitting between the baryons and the DM is needed to make these kinematically possible. Whether the coupling is primarily to e , μ , or τ depends on the relative Yukawa strengths in y' , but will therefore be the same across all baryonic states.

Challenges to the model. Having invented a specific model, we need to check that no important new annihilation channels were introduced, that were not present when we assumed the dark matter interacted only through its magnetic moment coupling. Due to the strong interactions of the DM constituents, there are indeed new possible diagrams shown in fig. 4 that can be problematic.

The first one, fig. 4(a), illustrates the process $\chi_1 \chi_1 \rightarrow \tilde{\eta}_i \gamma$ (where $i = \psi, S$) which involves a Yukawa interaction

$$\tilde{y}_i \tilde{\eta}_i \bar{\chi}_1 \chi_2 \quad (13)$$

in the effective theory below the confinement scale. This vertex only requires the rearrangement of the constituents into different bound states (with the annihilation of SS^* or $\psi\bar{\psi}$), so there is no *a priori* reason for it to be suppressed, and the couplings \tilde{y}_i might be relatively large. Thus this channel will dominate over that of fig. 1(a) if it is kinematically allowed, and one therefore needs to have $2m_{\chi_1} < m_{\tilde{\eta}_i}$. This can be accomplished by judicious choices of the mass matrix elements in (10). For example, with $m_y/\Lambda = 0.55$ and $m_\chi/\Lambda = 1.5$, we find that $m_{\chi_3} : m_{\chi_2} : m_{\chi_1} = 2.1 : 1 : 0.43$, which is sufficient. And the transition dipole moments are not much suppressed: $\mu_{23}/\mu_\eta = 0.6$, $\mu_{12}/\mu_\eta = 0.8$. Of course there will still be γ -ray continuum $\chi_1 \chi_1 \rightarrow 3\gamma$ annihilations due to the off-shell $\tilde{\eta}_i$, but these are phase-space suppressed, and the $\tilde{\eta}_i$ coupling to photons is also somewhat suppressed as we discuss presently.

The second diagram, fig. 4(b), requires an estimate of the $\tilde{\eta}_i$ coupling to two photons. To compute it from first principles would require knowledge of the nonperturbative matrix elements, but by comparing to the two-photon decays of η and η' in the real world, we estimate that the interaction takes the form

$$c \frac{q^2 e^2}{\Lambda} \tilde{\eta}_i F_{\mu\nu} F^{\mu\nu} \quad (14)$$

with $c \cong 0.1$, where $q = 1/2$ is the charge of the constituents in our model. The suppression by c is accompanied by an additional p -wave suppression due to the scalar coupling (as opposed to pseudoscalar) in (13). The cross section, ignoring interference with fig. 1(a), is

$$\sigma = \frac{2v^2}{\pi} \frac{c^2 \tilde{y}_i^2 q^4 e^4 m_\chi^4}{\Lambda^2 (4m_\chi^2 - m_{\tilde{\eta}_i}^2)^2} \quad (15)$$

Therefore this interaction turns out to be subdominant to that of fig. 1(a) unless it happens to be very close to resonance.

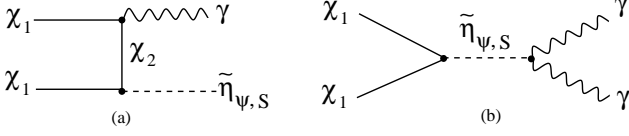


FIG. 4: New channels for DM annihilation present in the strongly interacting model, that should not dominate over the magnetic moment diagram of fig. 1(a).

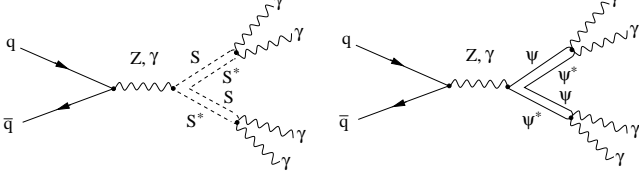


FIG. 5: Production and decay of $\tilde{\eta}_S$ and $\tilde{\eta}_\psi$ pairs at a hadron collider.

Collider signatures. Past studies of the collider signatures of magnetically interacting dark matter have focused on the production of the DM and its excited state $q\bar{q} \rightarrow \chi_1\chi_2$, mediated by s -channel γ or Z and the transition moment μ_{12} of χ_1 - χ_2 . The subsequent decay $\chi_2 \rightarrow \chi_1\gamma$ through the dipole moment can produce a hard monophoton that would pass experimental cuts [56, 57] if the χ_1 - χ_2 mass splitting exceeds 125-150 GeV. Otherwise the pair-production of DM can be searched for in a more generic way, through missing energy and initial state radiation of a hadronic monojet [58]-[61].

However the previous considerations assume that the dipole interaction remains hard at LHC energies. We have shown that whether one invokes a loop effect or more plausibly compositeness to generate the large moment needed to explain the 130 GeV line, it arises from physics well below the TeV scale, and therefore the mag-

netic moment effective description is not valid at high energies. It will open up to reveal the constituent particles rather than behaving like a dipole moment.

In the model we have proposed, the primary process will be to produce $\psi\bar{\psi}$ and SS^* pairs via virtual photon and Z . These will then hadronize into the mesons and baryons listed in table I. The final states will include the χ_1 and χ_2 (mostly η) dark matter particles, producing monophotons and monojets. But other composite states will also be produced, and they can have very different signatures because of their visible decays. For example, the bosonic bound states $\tilde{\eta}_S$ and $\tilde{\eta}_\psi$ can decay into two photons by direct annihilation of their constituents, as shown in fig. 5. Thus a very clean signal of two diphoton pairs, each of which has the same invariant mass $\gtrsim 260$ GeV, is predicted. The production of these particles is suppressed near threshold because of the overall charge neutrality of the mesons, leading to *e.g.*, destructive interference in the photon+ Z coupling to ψ and to $\bar{\psi}$. But at center-of-mass energies well above the particle masses, the vector boson resolves the constituents and couples fully to both of them. The spin and color summed/averaged partonic cross section for $\psi\bar{\psi}$ production from up quarks, at leading order in $1/s$, is given by⁶

$$\sigma \cong \frac{4\pi\alpha^2 Q_\psi^2}{9s \cos^4 \theta_w} \left(\frac{4}{9} + \frac{1}{36} \right) \sim 6 \text{ fb} \quad (16)$$

where for the numerical estimate we take $Q_\psi = 1/2$ and $s = 1$ TeV. Of course a serious prediction requires weighting by parton distribution functions, which will reduce (16) by around an order of magnitude because of the need to get the antiquark out of the sea, but we leave more detailed study for later. This estimate shows that the production is at a potentially interesting level for discovery at the LHC.

Even more interesting perhaps is the pair production of charged technibaryons, especially the N^- , which can decay via $N^- \rightarrow e\gamma$, or $N^- \rightarrow e\tilde{\eta}_{S,\psi}$ if the latter is kinematically allowed. Unlike the neutral pairs, here the production is not suppressed at threshold because both constituents have charges of the same sign. If both channels are open, the possible final states are $e\bar{e}$ plus two, three or four photons, due to the subsequent decays of the $\tilde{\eta}_{S,\psi}$ mesons. This is illustrated in fig. 6. However it is quite possible that the $N^- \rightarrow e\tilde{\eta}_{S,\psi}$ decays are kinematically blocked, in which case only the $e\bar{e}\gamma\gamma$ channel is present. Moreover, there is no reason to suppose that the y' coupling in (8) is only to electrons, and it may be natural to assume that the dominant coupling is to τ leptons, which would be more difficult to identify in the final state.

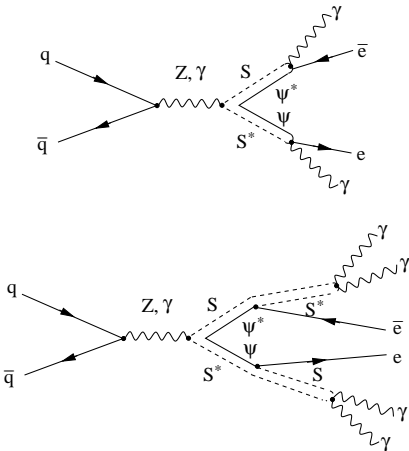


FIG. 6: Production of N^\pm pairs and two possible decay channels (third channel, $e\bar{e}\gamma\gamma$ not shown).

⁶ 4/9 and 1/36 are the squared hypercharges of the right and left components of the up quark.

Finally, there will be pair production of the charged bosonic technibaryons \tilde{N}_μ^\pm and \tilde{N}_ψ^\pm , which decay into leptons and dark matter. This however has a large standard model background since such events could be mistaken for W boson decays.

Conclusions. If the 130 GeV gamma ray line and the tentative accompanying line at 114 GeV are indeed due to dark matter annihilation into $\gamma\gamma$ and γZ , it is a challenge to explain its relatively strong annihilation into two monoenergetic photons, while keeping its annihilation into other particles that produce a continuum of secondary photons within bounds. We observed that a DM species comprising just 10-15% of the total DM mass density, possessing a transition magnetic moment of $\sim 2/\text{TeV}$ and a mass splitting $\gtrsim 10$ GeV can satisfy these requirements, with its relic density determined by the annihilations into photons and Z bosons. (Although an electric dipole moment of the same strength and smaller mass splitting could do the same, and be relevant for direct detection, we find it theoretically difficult to generate such a large EDM without an accompanying MDM of the same size.) This scenario leaves open the possibility of direct detection of some other, dominant DM species.

We further observed that it is difficult to explain a dipole moment of the needed magnitude through a loop effect, and much more natural to do so through a composite DM particle. We presented a simple model that accomplishes this with an electrically charged “quark” and scalar “quark” of a new $\text{SU}(2)$ gauge interaction that confines at the 100 GeV scale, and whose neutral mesonic bound state mixes with an elementary fermion to make the dark matter and its excited state. Remarkably, the magnetic moment is determined by the charge and mass of the constituents, leaving relatively little room for adjustment given that the DM has mass 130 GeV, yet coming out to approximately the desired value. We find the economy and predictiveness of the model striking.

Moreover the model makes an unambiguous prediction for hadron collider production of pairs of “unflavored” scalar-scalar or fermion-fermion bound states of mass $\gtrsim 130$ GeV, whose only decay channel is into two photons, and whose production cross section is estimated to be within reach of the LHC. This double diphoton pair signal comes in addition to the more discussed signature of DM pair production with missing energy and monophotons or monojets. And due to the pair production of charged bound states, further exotic final states can appear where two photons and one lepton (or three leptons in a related model) have the invariant mass of the charged parent. Although there is some freedom in varying the details that accommodate the decays of the charged bound states (necessary in order to avoid highly constrained charged relics), the two-photon decay of the neutral bound states is a robust prediction in any model that uses compositeness to generate large magnetic moments.

The dark matter interpretation of the 130 GeV line is expected to be tested definitively by upcoming gamma ray experiments [64], in particular by HESS-II which begins very soon. It is exciting to consider that complementary indirect evidence could come from the LHC on a similar time scale, if a new peak at mass $\gtrsim 260$ GeV (according to the discussion below eq. (13)), coming from pairs of diphoton events, should be discovered.

Acknowledgments. We thank M. Cirelli, M. Lüscher, V. Sanz, G. Servant, J. Shigemitsu, R. Teuscher, M. Trott, and C. Weniger for helpful discussions or communication. JC thanks the CERN theory division for its kind hospitality during the commencement of this work. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC).

Appendix A: $\chi_1\chi_2 \rightarrow f\bar{f}, WW, hZ$ coannihilation

The annihilation cross section for $\chi_1\chi_2 \rightarrow f\bar{f}$ for a standard model fermion pair at low velocity, via a magnetic moment interaction, can be expressed as

$$\sigma_{f\bar{f}}v = \alpha\mu^2 \sum_i N_i \quad (\text{A1})$$

where $N_i = 1$ for a hypothetical particle with unit electric charge and no coupling to the Z . The sum is over all kinematically accessible final states. For standard model fermions, and similarly parametrizing the contributions from W^+W^- and Zh final states, N_{eff} is

$$N_i = \begin{cases} q_i^2(1 - 2v_it_w\xi + (v_i^2 + a_i^2)t_w^2\xi^2), & f\bar{f} \\ \frac{1}{16}\left(\frac{m_Z}{m_W}\right)^4 \xi^2 \psi_{WW}, & WW \\ \frac{1}{16c_W^4} \xi^2 \psi_{hZ}, & hZ \end{cases} \quad (\text{A2})$$

The $f\bar{f}$ result is as given in ref. [38], where q_i is the electric charge, v_i, a_i are the vector and axial-vector couplings respectively to the Z , in units of q_e , and $\xi = (1 - m_Z^2/4(m_\chi^2))^{-1}$. For WW we define $\psi_{WW} = (1 + 4\epsilon_w - \frac{17}{4}\epsilon_w^2 - \frac{3}{4}\epsilon_w^3)(1 - \epsilon_w)^{1/2}$ with $\epsilon_w = m_W^2/m_\chi^2$. We find that the contribution from neutrinos is $\sum_i N_i = 3\xi^2/(8c_W^4)$, and for all charged leptons or quarks $v_i = t_w((4s_W^2|q_i|)^{-1} - 1)$, $a_i = t_w/(4s_W^2|q_i|)$, giving $\sum_i N_i = 10.4$ from fermionic final states, assuming $m_\chi = 130$ GeV and $s_W^2 = 0.23$. Because of a strong cancellation between the virtual γ and Z contributions to the WW channel, it contributes only 0.20 to $\sum_i N_i$. For hZ , we define $\psi_{hZ} = (1 - \frac{1}{2}(\epsilon_h - 5\epsilon_z) + \frac{1}{16}(\epsilon_h - \epsilon_z)^2) \times (1 - \frac{1}{2}(\epsilon_h + \epsilon_z) + \frac{1}{16}(\epsilon_h - \epsilon_z)^2)^{1/2}$ with $\epsilon_h = m_h^2/m_\chi^2$, contributing 0.22 to $\sum_i N_i$. Hence the total is $\sum_i N_i = 10.8$.

For the $f\bar{f}$ channels, the result from an electric dipole moment interaction is the same, except for an additional velocity-squared suppression factor of $v^2/3$.

Appendix B: $\chi_1\chi_1 \rightarrow \gamma\gamma, \gamma Z, ZZ$ annihilation

Defining $r = m_{\chi_2}/m_{\chi_1}$ and $\epsilon_Z = \frac{1}{4}(m_Z/m_{\chi_1})^2$ (superseding the definition of ϵ_Z in appendix A), the cross sections (times relative velocity) for $\chi_1\chi_1$ to annihilate into $\gamma\gamma, \gamma Z$ and ZZ are given by

$$\langle\sigma v\rangle = \frac{\mu^4 m_{\chi_1}^2}{4\pi} \begin{cases} \frac{4r^2}{(1+r^2)^2}, & \gamma\gamma \\ 2t_w^2 \frac{(1-\epsilon_Z)^3(r+\epsilon_Z)^2}{\left(\frac{1}{2}(1+r^2)-\epsilon_Z\right)^2}, & \gamma Z \\ t_w^4 \frac{(1-4\epsilon_Z)^{3/2}(r+2\epsilon_Z)^2}{\left(\frac{1}{2}(1+r^2)-2\epsilon_Z\right)^2}, & ZZ \end{cases} \quad (\text{B1})$$

The Dirac case corresponds to $r = 1$.

Appendix C: Explicit 3×3 DM mixing.

The parity symmetric 3×3 Majorana mass matrix M given in (10) can be diagonalized exactly, by converting it to a block diagonal $1 \times 1 \oplus 2 \times 2$ matrix. The exact mass eigenvalues are $-m_\eta$, which corresponds to an eigenvector entirely in the subspace of composite particles, and

$$\frac{1}{2} \left[m_\chi + m_\eta \pm \sqrt{(m_\chi - m_\eta)^2 + 8m_y^2} \right] \quad (\text{C1})$$

with eigenvectors mixing χ and the composite states. The physical masses are then as in equation (12) with

$$\delta m \equiv \frac{1}{2}(m_\eta - m_\chi) \left[1 - \sqrt{1 + 8m_y^2/(m_\chi - m_\eta)^2} \right]. \quad (\text{C2})$$

Notice that $\delta m > 0$ when $m_\chi > m_\eta$, leading to level repulsion.

We can now write the three eigenvectors as $\chi_1 = \cos\theta \eta_2 + \sin\theta \chi$, $\chi_2 = \eta_1$, and $\chi_3 = \cos\theta \chi - \sin\theta \eta_2$ in order of increasing mass and promote them to 4-component Majorana spinors. Then the transition dipole moment between the two composite states becomes

$$\mu_\eta \bar{\eta}_1 \sigma_{\mu\nu} \eta_2 F^{\mu\nu} = \mu_\eta \bar{\chi}_2 \sigma_{\mu\nu} (\cos\theta \chi_1 - \sin\theta \chi_3) F^{\mu\nu}. \quad (\text{C3})$$

The transition moment between the two lightest states is therefore suppressed by the cosine of the mixing angle θ , and there is no transition moment between the lightest and heaviest states.

Finally, explicitly finding the eigenvectors yields a mixing angle

$$\cos\theta = 2m_y / \sqrt{2\delta m^2 + 4m_y^2}. \quad (\text{C4})$$

In terms of $m_\chi, m_\eta, \delta m$, this becomes

$$\cos\theta = \sqrt{\frac{\delta m + m_\chi - m_\eta}{2\delta m + m_\chi - m_\eta}}. \quad (\text{C5})$$

A straightforward further algebraic rearrangement is necessary to write this in terms of m_χ/m_η and the relative mass splitting $\delta m/(m_\eta - \delta m)$ as in figure 3.

-
- [1] W. B. Atwood *et al.* [LAT Collaboration], *Astrophys. J.* **697**, 1071 (2009) [arXiv:0902.1089 [astro-ph.IM]].
 - [2] T. Bringmann, X. Huang, A. Ibarra, S. Vogl and C. Weniger, arXiv:1203.1312 [hep-ph].
 - [3] C. Weniger, arXiv:1204.2797 [hep-ph].
 - [4] E. Tempel, A. Hektor and M. Raidal, arXiv:1205.1045 [hep-ph].
 - [5] M. Su and D. P. Finkbeiner, arXiv:1206.1616 [astro-ph.HE].
 - [6] A. Rajaraman, T. M. P. Tait and D. Whiteson, arXiv:1205.4723 [hep-ph].
 - [7] A. Hektor, M. Raidal, E. Tempel, arXiv:1207.4466
 - [8] M. Su and D. P. Finkbeiner, arXiv:1207.7060 [astro-ph.HE].
 - [9] D. Hooper and T. Linden, arXiv:1208.0828 [astro-ph.HE].
 - [10] N. Mirabal, arXiv:1208.1693 [astro-ph.HE].
 - [11] A. Hektor, M. Raidal and E. Tempel, arXiv:1208.1996 [astro-ph.HE].
 - [12] W. Buchmuller and M. Garny, arXiv:1206.7056 [hep-ph].
 - [13] J. Goodman, M. Ibe, A. Rajaraman, W. Shepherd, T. M. P. Tait and H. -B. Yu, *Nucl. Phys. B* **844**, 55 (2011) [arXiv:1009.0008 [hep-ph]].
 - [14] M. Ackermann *et al.* [LAT Collaboration], arXiv:1205.2739 [astro-ph.HE].
 - [15] M. R. Buckley and D. Hooper, arXiv:1205.6811 [hep-ph].
 - [16] A. Geringer-Sameth and S. M. Koushiappas, arXiv:1206.0796 [astro-ph.HE].
 - [17] T. Cohen, M. Lisanti, T. R. Slatyer and J. G. Wacker, arXiv:1207.0800 [hep-ph].
 - [18] I. Cholis, M. Tavakoli and P. Ullio, arXiv:1207.1468 [hep-ph].
 - [19] X. -Y. Huang, Q. Yuan, P. -F. Yin, X. -J. Bi and X. -L. Chen, arXiv:1208.0267 [astro-ph.HE].
 - [20] S. Profumo and T. Linden, arXiv:1204.6047 [astro-ph.HE].
 - [21] A. Boyarsky, D. Malyshev and O. Ruchayskiy, arXiv:1205.4700 [astro-ph.HE].
 - [22] A. Ibarra, S. Lopez Gehler and M. Pato, arXiv:1205.0007 [hep-ph].
 - [23] L. Feng, Q. Yuan and Y. -Z. Fan, arXiv:1206.4758 [astro-ph.HE].
 - [24] R. -Z. Yang, Q. Yuan, L. Feng, Y. -Z. Fan and J. Chang, arXiv:1207.1621 [astro-ph.CO].
 - [25] I. Oda, arXiv:1207.1537 [hep-ph].
 - [26] M. T. Frandsen, U. Haisch, F. Kahlhoefer, P. Mertsch

- and K. Schmidt-Hoberg, arXiv:1207.3971 [hep-ph].
- [27] E. Dudas, Y. Mambrini, S. Pokorski and A. Romagnoni, arXiv:1205.1520 [hep-ph].
 - [28] J. M. Cline, arXiv:1205.2688 [hep-ph].
 - [29] K. -Y. Choi and O. Seto, arXiv:1205.3276 [hep-ph].
 - [30] B. Kyae and J. -C. Park, arXiv:1205.4151 [hep-ph].
 - [31] H. M. Lee, M. Park and W. -I. Park, arXiv:1205.4675 [hep-ph].
 - [32] B. S. Acharya, G. Kane, P. Kumar, R. Lu and B. Zheng, arXiv:1205.5789 [hep-ph].
 - [33] X. Chu, T. Hambye, T. Scarna and M. H. G. Tytgat, arXiv:1206.2279 [hep-ph].
 - [34] Z. Kang, T. Li, J. Li and Y. Liu, arXiv:1206.2863 [hep-ph].
 - [35] D. Das, U. Ellwanger and P. Mitropoulos, arXiv:1206.2639 [hep-ph].
 - [36] J. -C. Park and S. C. Park, arXiv:1207.4981 [hep-ph].
 - [37] C. B. Jackson, G. Servant, G. Shaughnessy, T. M. P. Tait and M. Taoso, JCAP **1004**, 004 (2010) [arXiv:0912.0004 [hep-ph]].
 - [38] N. Weiner and I. Yavin, arXiv:1206.2910 [hep-ph].
 - [39] S. Tulin, H. -B. Yu and K. M. Zurek, arXiv:1208.0009 [hep-ph].
 - [40] K. Sigurdson, M. Doran, A. Kurylov, R. R. Caldwell and M. Kamionkowski, Phys. Rev. D **70**, 083501 (2004) [Erratum-ibid. D **73**, 089903 (2006)] [astro-ph/0406355].
 - [41] S. Profumo and K. Sigurdson, Phys. Rev. D **75**, 023521 (2007) [astro-ph/0611129].
 - [42] J. H. Heo, Phys. Lett. B **702**, 205 (2011) [arXiv:0902.2643 [hep-ph]].
 - [43] E. Masso, S. Mohanty and S. Rao, Phys. Rev. D **80**, 036009 (2009) [arXiv:0906.1979 [hep-ph]].
 - [44] S. Chang, N. Weiner and I. Yavin, Phys. Rev. D **82**, 125011 (2010) [arXiv:1007.4200 [hep-ph]].
 - [45] B. Feldstein, P. W. Graham and S. Rajendran, Phys. Rev. D **82**, 075019 (2010) [arXiv:1008.1988 [hep-ph]].
 - [46] V. Barger, W. -Y. Keung and D. Marfatia, Phys. Lett. B **696**, 74 (2011) [arXiv:1007.4345 [hep-ph]].
 - [47] T. Banks, J. -F. Fortin and S. Thomas, arXiv:1007.5515 [hep-ph].
 - [48] A. L. Fitzpatrick and K. M. Zurek, Phys. Rev. D **82**, 075004 (2010) [arXiv:1007.5325 [hep-ph]].
 - [49] S. Patra and S. Rao, arXiv:1112.3454 [hep-ph].
 - [50] J. H. Heo and C. S. Kim, arXiv:1207.1341 [astro-ph.HE].
 - [51] M. Sher and S. Nie, Phys. Rev. D **65**, 093018 (2002) [hep-ph/0201220].
 - [52] H. Georgi, "Weak Interactions And Modern Particle Theory," Menlo Park, USA: Benjamin/Cummings (1984) 165p
 - [53] J. L. Goity and M. Sher, Phys. Lett. B **346**, 69 (1995) [Erratum-ibid. B **385**, 500 (1996)] [hep-ph/9412208].
 - [54] D. Chang and W. -Y. Keung, Phys. Lett. B **389**, 294 (1996) [hep-ph/9608313].
 - [55] D. S. M. Alves, S. R. Behbahani, P. Schuster and J. G. Wacker, Phys. Lett. B **692**, 323 (2010) [arXiv:0903.3945 [hep-ph]].
 - [56] S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1204.0821 [hep-ex].
 - [57] ATLAS collaboration, ATLAS-CONF-2012-085, 2012
 - [58] Y. Bai, P. J. Fox and R. Harnik, JHEP **1012**, 048 (2010) [arXiv:1005.3797 [hep-ph]].
 - [59] J. Goodman, M. Ibe, A. Rajaraman, W. Shepherd, T. M. P. Tait and H. -B. Yu, Phys. Lett. B **695**, 185 (2011) [arXiv:1005.1286 [hep-ph]].
 - [60] J. Goodman, M. Ibe, A. Rajaraman, W. Shepherd, T. M. P. Tait and H. -B. Yu, Phys. Rev. D **82**, 116010 (2010) [arXiv:1008.1783 [hep-ph]].
 - [61] J. -F. Fortin and T. M. P. Tait, Phys. Rev. D **85**, 063506 (2012) [arXiv:1103.3289 [hep-ph]].
 - [62] W. Lerche and D. Lust, Nucl. Phys. B **244**, 157 (1984).
 - [63] J. F. Nieves, Phys. Rev. D **35**, 1989 (1987).
 - [64] L. Bergstrom, G. Bertone, J. Conrad, C. Farnier and C. Weniger, arXiv:1207.6773 [hep-ph].